Name: ______________________

MoMathlon 2020

1. One day a redwood tree said, “I have been alive during all or part of seven centuries.” Rounded to the nearest year, what is the youngest that this tree could be?

2. Count on the fingers of your right hand, starting with 1 at your thumb, 2 at your pointer finger, 3 your middle finger, 4 your ring finger, 5 your pinky, then move back with 6 your ring finger, 7 your middle finger, 8 your pointer, etc. If the fingers are labeled thumb=1, pointer=2, middle=3, ring=4, pinky=5, which finger will 2020 hit?

3. Find the largest integer less than 100 which has exactly 6 positive divisors (including 1 and the number itself).

4. State the number of zeros that appear at the end of $2020!$.

5. A boy has as many brothers as he has sisters. His sister has twice as many brothers as she has sisters. How many children are in the family?

6. A 5x5x5 cube is built out of 1x1x1 cubes. If the outside surface of the large cube is painted, how many of the small cubes will have paint on exactly one face?

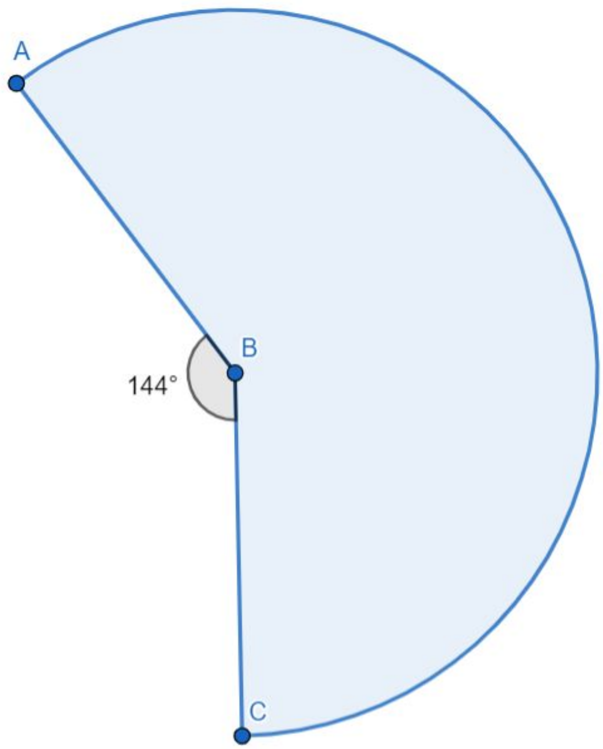
An NxNxN cube (N>2) is built out of 1x1x1 cubes. If the outside surface of the large cube is painted, how many of the small cubes will have paint on exactly one face? Express your answer in polynomial form.

7. Lola begins a spiral journey by walking one mile east, then two miles north, then three miles west, then four miles south, then five miles east, etc. After she has walked 55 miles, what is the square of the distance from her location to her starting point?

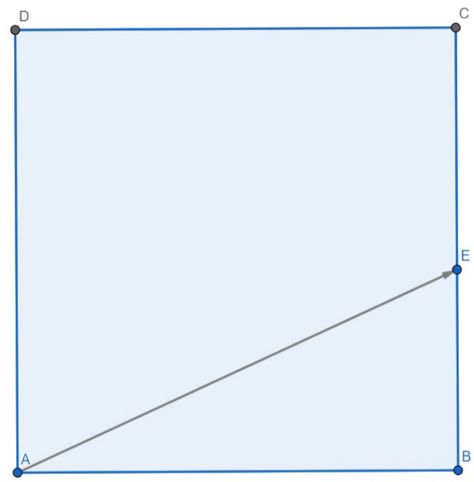
8. If a queen-and-a-quarter quotes a quail-and-a-quarter in a quarter of a day, how many quails can a queen quote in a day?

9. A bull is coming directly towards me at 30 mph. I consider two strategies: I consider running directly away from the bull at a fixed rate of speed, or running directly towards the bull with the same rate of speed. I find that with the first strategy, it will take the bull twice as long to reach me as with the second strategy. What is my running speed in mph?

10. A right circular cone is constructed from the circular sector shown below, by carefully aligning AB with CB, so that point B becomes the apex of the cone. If AB = 55, what is the height of the cone?



11. A mathematical billiard ball (i.e., a point with zero radius) is shot from corner A of a square of side length 52, and it hits side BC at point E, where BE = 17. The billiard then continues to bounce off the walls, until it returns to a corner. How many times will it bounce?

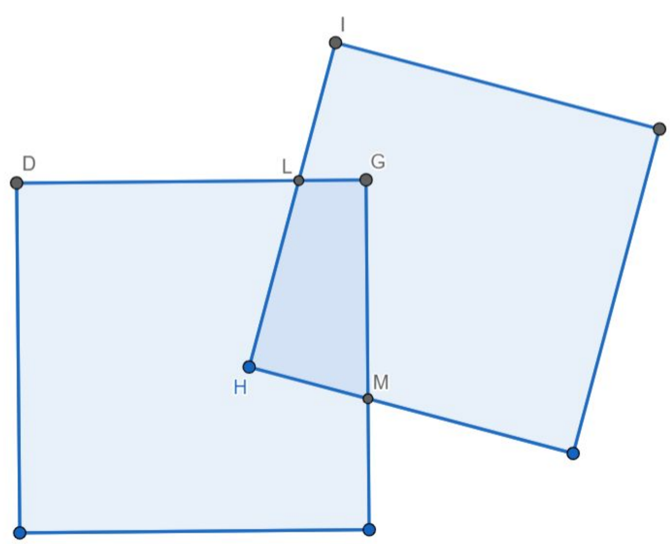


12. How many 4-digit base-10 integers have the property that the digits are increasing from left to right?

13. Find the smallest k such that F_k, the k^{th} Fibonacci number, is a multiple of 1001.

14. Find the remainder when $(1! + 2! + 3! + ... + 2019! + 2020!)^{2020}$ is divided by 6.

15. Consider two congruent squares, one of whose vertices lies in the center *H* of the other square. If $DG = 24$ and $\measuredangle ILD = 108°$, find the area of quadrilateral *LGMH*.

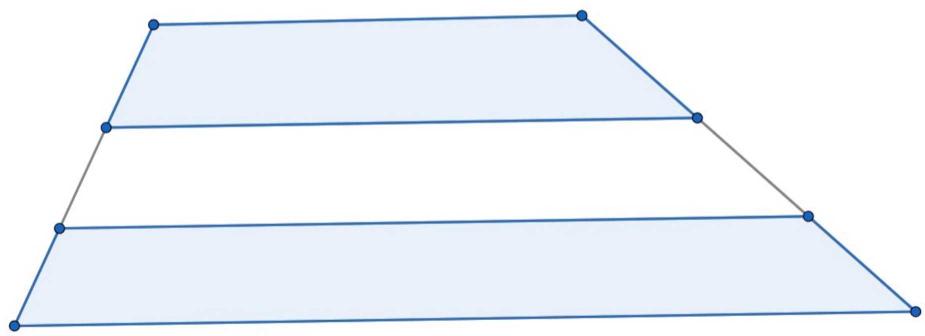


16. Triangle *ABC* has side lengths $AB = 5$, $BC = 12$, and $AC = 13$. A circle with center D is inscribed in this triangle, and is tangent to side AB at the point E. Find the area of triangle AED.

17. The numbers 1,2,3,4,5 are written on a blackboard on Monday morning. Each night, someone comes up to the blackboard and picks two numbers, and erases them, replacing them with the sum of their sum and their product (for example, the numbers 1 and 5 would be replaced with $1 \cdot 5 + 1 + 5 = 11$.) By Thursday night, there will just be one number left. Let L be the largest possible value for this number, and let *S* be the smallest possible value. Find $L + S$.

18. Find the largest number that is the product of positive integers whose sum is 22.

19. A trapezoid has a height of 10 with bases of length 23 and 32. Lines are drawn parallel to the bases to divide the trapezoid into 11 strips of equal thickness; the figure below depicts the situation with just 3 stripes. If every other stripe, starting from the top, is shaded, find the total area of the shaded stripes.



20. A sock drawer has a huge supply of red, white, and blue socks. You are taking socks out of the drawer in the dark, and cannot see the colors. What is the least number of socks that you must take in order to guarantee that you have taken at least 8 red socks or at least 6 white socks or at least 9 blue socks?

21. Five numbers are in arithmetic progression; in other words, each number in the sequence is equal to the previous number plus a constant. For example, 7, 11, 15, 19 are four numbers in an arithmetic progression. If the first number is 2020 and the third number is 2, what is the sum of the five numbers?

22. Let $\frac{a}{b}$ be the sum, in lowest terms, of

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + ... + \frac{1}{2018 \cdot 2019} + \frac{1}{2019 \cdot 2020}$$

Find $a + b$.

23. How many positive integers between 1 and 1001, inclusive, are relatively prime to 1001?

24. Let ABCDE be a 5-digit number. When EDCBA is subtracted from ABCDE, the result is a five-digit number whose first four digits are 3, 4, 9, 4. What is the final digit?

25. Here are some of the entries in a multiplication table that goes up to 12×12:

	1	2	$\cdots$	12
1	1	2	$\cdots$	12
2	2	4		24
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
12	12	24	$\cdots$	144

Find the sum of all the entries. Do not include the labels in the top row or the left column.

26. The tetrakaidecahedron is a polyhedron composed of 14 faces: 6 squares and 8 regular hexagons, At each vertex, two hexagons and one square meet. How many vertices does this polyhedron have?